Flat-Plate Solar Collectors

The Absorption of Solar Radiation

A flat-plate solar collector usually has a non-selective or a selective black plate with one or two glass covers a few centimeters above the black plate, and a well insulated back. The length of the plate is typically about 2 m. Edge effects are usually small.

The transmittance $\tau(\theta)$ of a glass cover for solar radiation depends on the angle of incidence θ. Typical values for clear glass are given in Table 1.

Table 1. Transmittance of a Glass Cover.

θ:	0°	60°	70°	80°	90°
$\tau(\theta)$:	0.9	0.8	0.65	0.35	0

The absorptance $\alpha(\theta)$ of the black plate for solar radiation also depends on the angle of incidence θ. Table 2 shows typical values for $\alpha(\theta)$ and the product $\tau(\theta)\alpha(\theta)$.

Table 2. Absorptance of a Black Plate.

θ:	0°	60°	70°	80°	90°
$\alpha(\theta)$:	0.92	0.85	0.75	0.60	0
$\tau(\theta)\alpha(\theta)$:	0.83	0.68	0.49	0.21	0

The solar irradiance I_{in} incident on the cover glass is given by

$$I_{in} = I_b \cos\theta + I_d,$$

where I_b is the beam solar irradiance, θ is the angle of incidence, and I_d is the diffuse irradiance.

If there is one glass cover the solar irradiance on the black plate is

$$\tau(\theta) I_b \cos\theta + \tau_m I_d,$$

where τ_m is the mean value of $\tau(\theta)$. The solar radiation flux q_{abs} absorbed by the black plate is given by

$$q_{abs} = \tau(\theta)\alpha(\theta)\, I_b \cos\theta + (\tau\alpha)_m I_d,$$

where $(\tau\alpha)_m$ is the mean value of $\tau(\theta)\alpha(\theta)$. The mean value of $\tau(\theta)\alpha(\theta)$ can be found by means of integrals over the hemispherical sky as follows:

$$(\tau\alpha)_m = [\int_0^{\pi/2}\tau(\theta)\ \alpha(\theta)\ \sin\theta\ \cos\theta\ d(\theta)]\ /\ [\int_0^{\pi/2}\sin\theta\ \cos\theta\ d(\theta)].$$

For one glass cover the result is approximately $(\tau\alpha)_m = 0.70$.

Heat Losses

The glass cover behaves nearly as a black body for long-wave radiation. We can assume that the emittance ε_c of the glass cover is 0.95.

The emittance ε_b of the black plate for long-wave radiation depends on whether the surface is non-selective or selective. Typically we have

$\varepsilon_b = 0.92$ for a non-selective surface,
$\varepsilon_b = 0.10$ for a selective surface.

We shall consider a collector with one glass cover. Let

T_a = ambient temperature,
T_b = black plate temperature,
T_c = glass cover temperature,

where absolute temperatures must be used for radiation calculations.

Heat is lost by conduction through the back insulation. It can be reduced to a low rate by inexpensive insulation materials. Typically the back loss might be given by the formula

$$h_{ba}(T_b - T_a),$$

where the heat transfer coefficient is $h_{ba} = 0.3\ W/m^2K$.

Heat is lost from the black plate to the glass cover by convection and radiation. Experience has shown that, for free convection the Nusselt number Nu in air spaces between parallel plates with Grashof numbers Gr in the range 10^4 to 10^7, we have

$Nu = 0.152\ Gr^{0.281}$ for horizontal plates,
$Nu = 0.093\ Gr^{0.310}$ for plates tilted at an angle 45 degrees.

Here

$$Gr = g\beta(T_b - T_c)L^3/\nu^2,$$

where we assume as a typical example for air:

g = acceleration of gravity = 9.8 m/s^2,
β = coefficient of thermal expansion = 1/T, T = 60°C = 333 K,
$T_b - T_c$ = 80°C − 40°C = 40 K,
L = spacing = 50 mm = 0.05 m,
ν = kinematic viscosity = 0.194×10^{-4} m^2/s.

This gives Gr = 3.91×10^5, which is within the range 10^4 to 10^7 mentioned above.

Assume a tilt angle 15°. Then we estimate, by interpolation, Nu = 0.132 $Gr^{0.291}$ = 5.572.

Also since

$$Nu = hL/k,$$

where

h = heat transfer coefficient,
L = 0.05 m,
k = thermal conductivity of air = 0.02750 W/mK,

we have h = 3.06 W/m^{2}K. This calculation shows that in general the heat transfer coefficient h is a function of T_b and T_c.

For the heat loss by radiation between the black plate and the glass cover we have the expression

$$[\sigma(T_b^4 - T_c^4)] / [\varepsilon_b^{-1} + \varepsilon_c^{-1} - 1] = \varepsilon_{bc}\sigma(T_b^4 - T_c^4),$$

where σ is the Stefan-Boltzman constant 56.7×10^{-9} W/m^{2}K^4.

Thus the total heat loss q_{ba} from the black plate can be written

$$q_{ba} = h_{ba}(T_b - T_a) + h_{bc}(T_b - T_c) + \varepsilon_{bc}\sigma(T_b^4 - T_c^4), \;.....(1)$$

where h_{bc} depends on T_b and T_c, and on the angle of tilt of the collector.

The heat loss from the glass cover to the surroundings must be the same, in the steady state, as the heat loss from the black plate to the glass cover. We have for the heat loss from the glass cover

$$q_{ca} = h_{ca}(T_c - T_a) + \varepsilon_c \sigma T_c^4 - \varepsilon_c L, \; .....(2)$$

where the convection heat transfer coefficient h_{ca} is difficult to estimate because it is partly due to free convection and partly due to forced convection by wind blowing over the collector. The following formula is recommended:

$$h_{ca} = 2.8 + 3.0V \text{ W/m}^2\text{K},$$

where V is the wind speed in meters per second. The second term $\varepsilon_c \sigma T_c^4$ in equation (2) is the long-wave radiation from the glass cover, and the third term $\varepsilon_c L$ is the long-wave radiation absorbed by the glass cover from the sky.

Collector Efficiency in the Steady State

We must calculate the temperatures T_b and T_c from the radiation fluxes and the ambient temperature T_a.

First assume a value for q_{ca} and solve equation (2) for T_c. Now equation (1) shows that

$$q_{ca} = h_{bc}(T_b - T_c) + \varepsilon_{bc} \sigma (T_b^4 - T_c^4), \; .....(3)$$

from the heat balance of the glass cover in the steady state. This equation is solved for T_b. Finally q_{ba} is found from equation (1).

Repeat the calculation for different values of q_{ca} to obtain q_{ba} as a function of T_b. This is easy to do numerically with a programmable calculator.

Let q_{out} be the heating power output of the collector per unit area. It can be varied within the feasible limits by controlling the operating conditions of the collector. The heat balance for the black plate gives

$$q_{abs} = q_{ba} + q_{out},$$

where q_{abs} depends on I_b, θ, and I_d; and the heat loss rate q_{ba} is known as a function of T_b. The **overall efficiency**

$$\eta = q_{out}/q_{in}$$

is therefore known as a function of the radiation fluxes and the black plate temperature T_b.

Simplifications in the Theory

In practical flat-plate solar collectors the temperature of the flat plate is not uniform. In tube-in-sheet designs the temperature of the sheet between the tubes is higher than the temperature of the tubes. Furthermore, the temperature of the tubes is higher at the outlet ends than at the inlet ends. The use of a single black plate temperature T_b is therefore a simplification.

Another simplification is to put

$$q_{abs} = \gamma q_{in},$$

where γ is the **optical efficiency**, which is equal to the mean transmittance-absorptance product $(\tau\alpha)_m$.

The heat balance equation is then written

$$q_{out} = \gamma q_{in} - U(T_b - T_a),$$

where U is the **overall heat loss coefficient** between the black plate and the surroundings. The efficiency is now given by

$$\eta = q_{out}/q_{in} = \gamma - U(T_b - T_a)/q_{in}.$$

Thus γ is the efficiency when $T_b = T_a$. Often U is assumed to be constant, and the **stagnation temperature** T_{max} obtained when no heat is extracted from the collector is estimated to be

$$T_{max} = T_a + \gamma q_{in}/U.$$

In reality, because of the non-linear relations between the heat losses and the temperature differences, U varies with $T_b - T_a$. With good approximation we can write

$$q_{out} = \gamma q_{in} - U_1(T_b - T_a) - U_2(T_b - T_a)^2$$

and

$$\eta = \gamma - U_1(T_b - T_a)/q_{in} - U_2(T_b - T_a)^2/q_{in},$$

where U_2 is small compared with U_1. The graph of η versus $(T_b - T_a)$ is slightly curved. (See Fig. 1.)

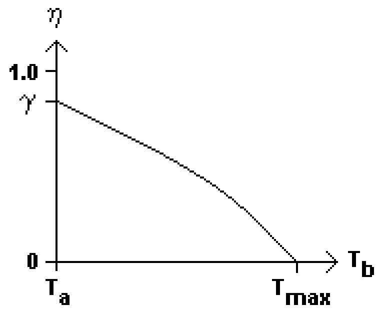

Fig. 1. The efficiency of a flat plate collector.

If the solar radiation falling on the collector changes rapidly, due to the passage of clouds, the collector will take time to change its temperature because of its heat capacity. This may be important and require separate analysis. The theory of non-steady state processes in solar collectors is very complicated, and is ignored in steady state calculations.

Practical Collector Performance Parameters

In practice it is convenient to use the fluid temperature T_f instead of the black plate temperature T_b. The total heat extraction rate Q_{out} from a collector of area A is then written

$$Q_{out} = AF'[\gamma q_{in} - U(T_f - T_a)], \quad .....(1)$$

where F' is called the **collector efficiency factor**. In good designs F' is nearly unity. F'γ is the effective transmittance-absorptance product; and F'U is the heat loss coefficient between the fluid and its surroundings.

We may define the thermal resistance R between the black plate and the fluid by the equation

$$q_{out} = (T_b - T_f)/R,$$

and derive equation (1) from the heat balance equation between the black plate and the surroundings. It is then found that

$$F' = 1/(1 + RU).$$

Consider a fluid with mass flow rate m and specific heat capacity c flowing a total distance L through a collector of area A. The heating of the fluid with respect to distance x through the collector is given by

$$mc(dT(x)/dx) = (AF'/L)[\gamma q_{in} - U(T(x) - T_a)].$$

Assume U is constant. Then this is a first order non-homogeneous linear differential equation. It can be solved to give the basic equation relating the inlet and outlet fluid temperatures T_{in} and T_{out} as follows:

$$[\gamma q_{in} - U(T_{in} - T_a)]\exp(-AF'U/mc) = [\gamma q_{in} - U(T_{out} - T_{in})]. \;.....(2)$$

Often the performance of a collector is written in terms of the fluid inlet temperature T_{in} and a **heat removal factor** F_R, as follows

$$Q_{out} = AF_R[\gamma q_{in} - U(T_{in} - T_a)]. \;.....(3)$$

Writing $Q_{out} = mc(T_{out} - T_{in})$, and eliminating T_{out} with the help of (2), shows that

$$F_R = (mc/AU)[1 - \exp(-AF'U/mc)]. \;.....(4)$$

For small flow F_R is small and the fluid temperature approaches the stagnation temperature T_{max}. For large flow F_R is large, but the rise in fluid temperature is small.

The Collector as a Heat Exchanger

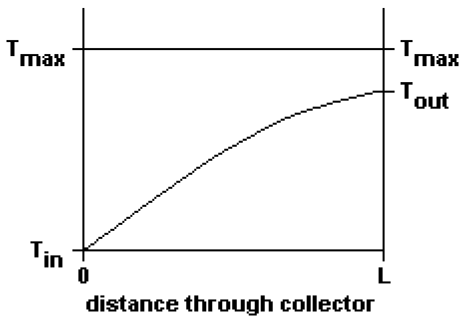

Fig. 2. The flat-plate collector as a heat exchanger.

A solar collector may be regarded as a conventional heat exchanger transferring heat from solar radiation at a constant temperature T_{max} to the collector fluid (see Fig. 2). First we define the **log mean temperature difference** LMTD by the equation

$$LMTD = (T_{out} - T_{in})/\log[(T_{max} - T_{in})/(T_{max} - T_{out})]. \;.....(5)$$

Then, putting $T_{max} = T_a + \gamma q_{in}/U$, and $Q_{out} = mc(T_{out} - T_{in})$, we obtain from equation (2):

$$Q_{out} = AF'U(LMTD). \;.....(6)$$

Another method uses the **heat exchanger effectiveness**

$$E = Q_{out}/Q_{max}, \quad .....(7)$$

where Q_{max} is the heat transfer rate when $T_{out} = T_{max}$.

Putting

$$Q_{out} = mc(T_{out} - T_{in}),$$
$$Q_{max} = mc(T_{max} - T_{in}),$$
$$T_{max} = T_a + \gamma q_{in}/U,$$

we obtain from equation (2):

$$E = 1 - \exp(-AF'U/mc).$$

Standard theory for this type of heat exchanger gives

$$E = 1 - \exp(-N), \quad .....(8)$$

where N is the **number of heat transfer units**. Therefore in the collector

$$N = AF'U/mc. \quad .....(9)$$

Note that (4), (8), and (9) give the following relation between the different measures of performance:

$$F_R N = F'E.$$

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