# Principles of Solar Thermal Conversion

## Conversion to Work

Heat from a solar collector may be used to drive a heat engine operating in a cycle to produce work. A heat engine may be used for such applications as water pumping and generating electricity.

The thermal output $Q_{out}$ of a concentrating collector operating at temperature T is given by

$$Q_{out} = F'[\gamma A_{in} q_{in} - U A_{abs}(T - T_a)],$$

where $A_{in}$ is the area of the incident solar radiation and $A_{abs}$ is the area of the absorber. (The other symbols are the same as in the other lecture notes.) The quantity $A_{in}/A_{abs}$ is called the **concentration ratio**. High concentration ratios are obtained by making $A_{in}$ the area of a system of mirrors designed to concentrate the solar radiation received onto a small absorber of area $A_{abs}$. Heat losses from the absorber are reduced by the smaller size of the absorber. Consequently, high concentration ratios give high collector temperatures. The stagnation temperature $T_{max}$ is given by:

$$\gamma A_{in} q_{in} = U A_{abs}(T_{max} - T_a).$$

For example, if the optical efficiency is $\gamma = 0.8$, the incident solar irradiation is $q_{in} = 800\ W/m^2$, the ambient temperature is $T_a = 30°C$, and the heat loss coefficient is $U = 10\ W/m^2K$, then a concentration ratio $A_{in}/A_{abs} = 1$ (no concentration) gives $T_{max} = 94°C$, and a concentration ratio $A_{in}/A_{abs} = 10$ gives $T_{max} = 670°C$.

The collector efficiency $\eta_c$ at operating temperature T is

$$\eta_c = Q_{out}/A_{in} q_{in} = F'[\gamma - U A_{abs}(T - T_a)/A_{in} q_{in}] = F'\gamma(T_{max} - T)/(T_{max} - T_a).$$

The available mechanical power from the thermal power output of the collector that would be obtained using a Carnot cycle is $Q_{out}(1 - T_a/T)$, where the temperatures are absolute temperatures.

The **second law efficiency** $\eta_2$ of a heat engine is defined by

$$\eta_2 = \text{(mechanical power delivered)}/\text{(available mechanical power)}.$$

Suppose a heat engine with second law efficiency $\eta_2$ uses as input the thermal power $Q_{out}$ from the solar collector. The first law efficiency of the engine is

$$\eta_1 = (\text{mechanical power delivered})/Q_{out} = \eta_2(1 - T_a/T),$$

and the first law efficiency $\eta$ of the system (collector plus heat engine) is

$$\eta = \eta_c \eta_1.$$

Now, given F', $\gamma$, $\eta_2$, $T_a$, and $T_{max}$, we can find the maximum efficiency obtainable, and the optimum operating temperature $T_{opt}$ from the condition $d(\eta)/dT = 0$. This occurs at the optimum temperature

$$T_{opt} = \sqrt{[T_{max} T_a]},$$

and the maximum efficiency is obtained by putting $T = T_{opt}$

For example, putting F' = 0.9, $\gamma = 0.8$, $\eta_2 = 0.6$, $T_a = 30°C = 303$ K, we get the efficiencies $\eta_{max}$ for different degrees of concentration shown in Table 1. Very low overall efficiencies are obtained unless operating temperatures greater than 500°C are used. Expensive concentrating systems are needed to reach these high temperatures, so commercial viability is difficult.

Table 1. Efficiencies for Converting Solar Radiation to Work

| $T_{max}$ | $T_{opt}$ | $\eta_{max}$ |
|---|---|---|
| 100°C | 63°C | 2.2% |
| 200°C | 106°C | 4.8% |
| 400°C | 179°C | 8.5% |
| 800°C | 297°C | 13.2% |
| 1600°C | 480°C | 18.4% |

# Domestic Water Heating

Suppose we wish to add heat $Q_r$ to a reservoir at temperature $T_r$ from a collector at temperature $T > T_r$. If the heating power from the collector is $Q_{out}$, the first law efficiency of the heat transfer process is

$$\eta_1 = Q_r/Q_{out}.$$

The available mechanical power in the heat extracted from the collector is

$$Q_{out}(1 - T_a/T).$$

The mechanical power that would be needed to operate a reversible heat pump delivering heat $Q_r$ to the reservoir at temperature $T_r$ from ambient temperature $T_a$ is

$$Q_r(1 - T_a/T_r).$$

Therefore, the second law efficiency $\eta_2$, defined as the mechanical power needed for heating divided by the available mechanical power, is given by

$$\eta_2 = Q_r(1 - T_a/T_r)/Q_{out}(1 - T_a/T) = \eta_1(1 - T_a/T_r)/(1 - T_a/T).$$

This has its maximum value $\eta_2 = \eta_1$ when $T = T_r$.

Collector efficiency increases as $T_{max}$ increases, but acceptable efficiencies at the temperatures 50°C to 60°C needed for domestic hot water systems are obtainable with flat-plate collectors.

Concentrating collectors producing heat at temperatures T considerably greater than $T_r$ give low second law efficiencies, and are uneconomical because of their high cost. The use of gas or oil burning at high flame temperatures for producing domestic hot water is also an inefficient waste of available energy.

These considerations show why domestic flat-plate solar water heaters are commercially successful.

# Refrigeration

We can use thermal solar energy to produce refrigeration by converting solar heat into mechanical power and using this power to drive a compression refrigerator. However, high collector temperatures are needed to generate the mechanical power with acceptable efficiency. Consequently, it is better to use an absorption refrigerator.

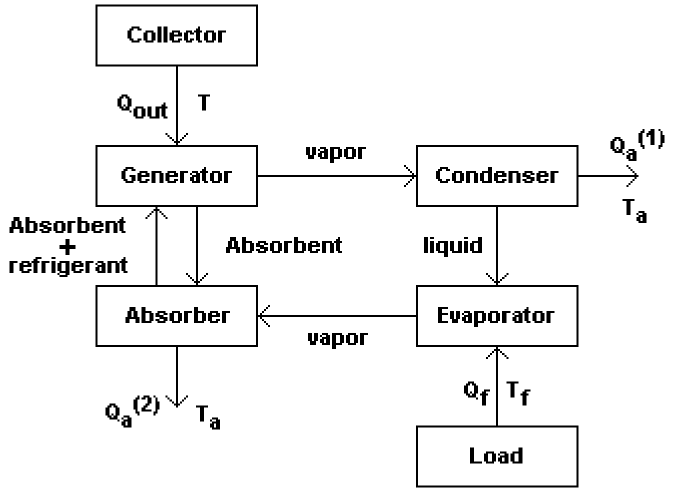


Fig. 1. An absorption refrigerator.

An absorption refrigerator uses a refrigerant (such as ammonia) and an absorbent (such as water). The cycle (Fig. 1.) consists of four parts:

1. The use of solar heat to drive off the refrigerant as a vapor from the liquid absorbent at the collector temperature T.
2. The condensation of the refrigerant, and the rejection of the heat of condensation at ambient temperature $T_a$.
3. The evaporation of the refrigerant at the refrigeration temperature $T_f$ with the extraction of heat from the refrigeration load.
4. The reabsorption of the refrigerant, and the rejection of the heat of adsorption at ambient temperature $T_a$.

Let $Q_{out}$ be the solar heating power at temperature T; let $Q_f$ be the refrigeration rate at temperature $T_f$; and let $Q_a$ be the heat rejection rate at temperature $T_a$. By the first law of thermodynamics, since energy is conserved,

$$Q_{out} + Q_f = Q_a.$$

By the second law of thermodynamics, since the entropy change in the cycle is zero,

$$(Q_{out}/T) + (Q_f/T_f) = Q_a/T_a.$$

Eliminating $Q_a$, we obtain the ideal cooling ratio

$$Q_f/Q_{out} = [1 - T_a/T]/[(T_a/T_f) - 1].$$

In real systems the actual refrigeration rate $Q_f'$ is less than ideal. We define the first law efficiency

$$\eta_1 = Q_f'/Q_{out},$$

and the second law efficiency

$$\eta_2 = Q_f'/Q_f = \eta[(T_a/T_f) - 1]/[1 - T_a/T].$$

The collector temperature T required for operating an absorption refrigerator is such that $T - T_a$ is slightly greater than $T_c - T_f$. For example, if $T_f = -10°C$ and $T_a = 30°C$, then T must be slightly greater than 80°C. This temperature can be obtained with flat-plate collectors.

The ideal cooling ratio in this example is 0.93. In practice a first law efficiency of 0.6 might be obtained. In the latter case the second law efficiency becomes 0.64. If the solar collector efficiency $\eta_c$ is 0.5, then the overall performance of the collector and absorption refrigerator is $\eta = \eta_c\eta_1 = 0.3$.

The coefficient of performance of a compression refrigerator is defined to be the cooling rate obtained divided by the mechanical power input. In practice this is about 3. Therefore, to obtain the same overall performance as an absorption system a solar collector and heat engine would need a first law efficiency of 0.1. This would require a solar operating temperature over 200°C and expensive concentrating collectors, which use only direct solar radiation. Therefore, absorption systems seem to be more promising for solar powered refrigeration, especially large scale use such as in food stores and air conditioning.

# Upgrading by Reversed Absorption

It is possible in principle to collect solar heat and raise the temperature of the thermal output by means of an absorption system working in reverse. Some of the heat collected must be sacrificed, but it is possible to reach temperatures beyond the collector stagnation temperature $T_{max}$. It is interesting to study the method, even though the absorption system may be expensive compared with the cost of high temperature collectors.

The cycle (Fig. 2) is similar to that of a solar absorption refrigeration system, except that the working temperatures are higher.

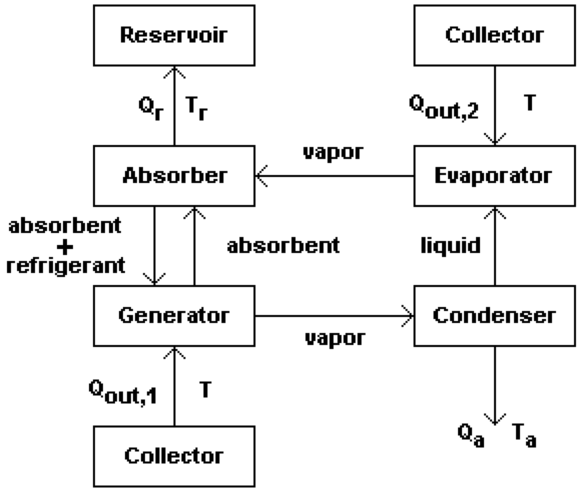


Fig. 2. A reversed absorption heat upgrader.

The solar heating power $Q_{out}$ at temperature T is used partly to provide the heat of generation $Q_{out,1}$ and partly to provide the heat of evaporation $Q_{out,2}$. Thus $Q_{out} = Q_{out,1} + Q_{out,2}$. The condensation of the refrigerant occurs at ambient temperature $T_a$ accompanied by the rejection of latent heat $Q_a$. The reabsorption of the refrigerant vapor produces heat $Q_r$ rejected to a reservoir at temperature $T_r$.

By the first law we have $Q_{out} = Q_r + Q_a$, and by the second law we have $Q_{out}/T = (Q_r/T_r) + (Q_a/T_a)$. Eliminating $Q_a$, we obtain the ideal heating ratio

$$Q_r/Q_{out} = (1 - T_a/T)/(1 - T_a/T_r).$$

As an example, suppose that $T_a = 30°C$, $T = 80°C$, and $T_r = 140°C$. Then the ideal heating ratio is 0.53. The actual heating ratio would be less than this.

# Thermodynamics of Solar Radiation

It can be shown that the thermodynamics of blackbody radiation in an enclosure in thermal equilibrium with the walls of the enclosure is completely described by the following equations, where $\alpha = 4\sigma/c$, $\sigma$ is the Stefan-Boltzmann constant, c is the velocity of light, and the other symbols have their usual meanings in thermodynamics.

$$P = (1/3)\alpha T^4,$$
$$U/V = \alpha T^4,$$
$$S/V = (4/3)\alpha T^3.$$

The pressure P, the internal energy per unit volume U/V, and the entropy per unit volume S/V are functions of temperature T only.

We are interested in the available mechanical power extractable from solar radiation. This is given by the **availability potential** A defined by

$$A = U + P_a V - T_a S,$$

where $P_a$ and $T_a$ are the pressure and temperature of the environment. The difference $A_f - A_i$ between a final and an initial state is the maximum available work from a closed thermodynamic system. For radiation we have

$$A/V = \alpha T^4[1 - (4/3)(T_a/T) + (1/3)(T_a/T)^4].$$

Solar radiation differs from blackbody radiation in two ways:

1. Bright sunlight is highly directional.
2. The intensity of the light is much less than that of blackbody radiation having the same spectrum.

The nature of solar radiation can be understood as follows:

Imagine the reversible expansion of blackbody radiation from a volume $V_1$ to a volume $V_2$ in an adiabatic enclosure whose walls are perfectly reflecting mirrors. The spectrum of the radiation can be kept the same as that of a blackbody by a black particle of matter inside the enclosure.

Since the entropy remains unchanged we have $S_2 = S_1$, so that $(T_2/T_1)^3 = V_1/V_2$, and the temperature decreases. The total energy $\alpha T^4 V$ also decreases because work is done by the radiation pressure during the expansion.

Isolated radiation can (in theory) be expanded irreversibly into a second radiation-free volume whose walls are perfect mirrors without doing work by removing a partition separating the two volumes. If there is no black particle of matter present, the spectrum remains the same, but the radiation occupies a larger volume. Consequently, it is no longer blackbody radiation because its spectrum and energy density do not correspond. As a result we cannot define a temperature for it.

However, if a black particle of matter is introduced to produce thermodynamic equilibrium within the expanded volume, then the spectrum changes to a blackbody spectrum. Since no energy has been exchanged with the environment, $U_2 = U_1$, and the temperature falls in accordance with the equation

$$(T_2/T_1)^4 = V_1/V_2.$$

Also the entropy is increased in accordance with the equation

$$S_2/S_1 = T_2/T_1.$$

Now imagine blackbody radiation escaping from a radiation enclosure through a pinhole into empty space. Its energy and entropy per unit solid angle do not change with distance because the rays can be reflected back to their source by an ideal spherical mirror. Since the ratio S/U per unit solid angle does not change, the temperature of the radiation does not change.

Extraterrestrial solar radiation is similar to radial blackbody radiation from a source at 5800 K after traveling a distance 150 million kilometers. Taking the temperature of the terrestrial environment as 300 K we obtain the availability potential per unit volume $A/V = 0.93U/V$. This explains why focusing direct solar radiation gives high temperatures and high first law thermodynamic efficiencies. Corrections are needed for the apparent size of the sun's disk, and for the attenuation of the radiation by the atmosphere.

If solar radiation is made diffuse by reflection at an ideally white surface, the spectrum remains unchanged but the radiation can no longer be focused. A pencil of solar radiation let into a hollow cavity with white diffusely reflecting walls still has the same energy and spectrum, but the energy density per unit solid angle has decreased and the entropy has increased. The effect is the same as if the radiation had been expanded without doing work. After the establishment of thermodynamic equilibrium with the help of a black particle of matter, the temperature is lowered and the entropy is further increased. It has been shown that the temperature of solar radiation after complete diffuse reflection is 1350 K. In this case the availability potential is given by $A/V = 0.70U/V$. This is of interest as the theoretical upper limit to the energy available for photosynthesis in plants.

## Upper Limits to the Conversion of Solar Energy

Suppose it is possible to trap solar radiation in an enclosed volume with perfectly reflecting walls at the temperature of the sun $T_s = 5800$ K. We can imagine this radiation to be cooled by transferring heat from it to a reversible heat engine operating between the temperature of the trapped radiation and the ambient temperature $T_a$. The work W obtained is then

$$A_s - A_a = \sigma T^4[1 - (4/3)T_a/T_s + (1/3)(T_a/T_s)^4].$$

If $T_a = 300$ K, then the efficiency is $W/\sigma.T^4 = 0.93$.

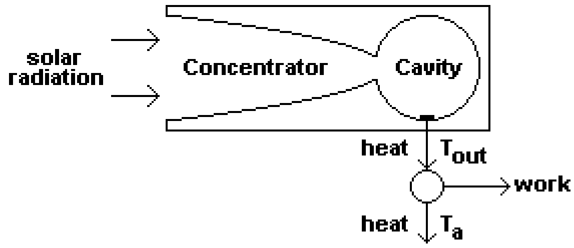


Fig. 3. An ideal solar thermal converter.

This is of academic interest only; but we may bear it in mind when considering the imaginary device shown in Fig. 3, which is potentially capable of achieving a high efficiency. It consists of an ideal compound parabolic concentrator that focuses direct solar radiation into a cavity whose walls are reflecting except for a blackbody receiver at a temperature $T_{out}$. The black receiver transfers heat to a reversible engine to produce work and reject heat at ambient temperature. The ideal efficiency of the heat engine is $(1 - T_a/T)$. The cavity is designed so that it exchanges radiation only with the disk of the sun, which is at a temperature $T_s$. The efficiency of collection is therefore $1 - (T/T_s)^4$. Accordingly, the overall efficiency of the system is

$$\eta = [1 - (T/T_s)^4][1 - T_a/T].$$

If $T_s = 5800$ K and $T_a = 300$ K, then the efficiency has a maximum $\eta = 0.85$ at an operating temperature $T = 2480$ K. This is the theoretical limit for the conversion of extraterrestrial solar radiation into mechanical work.

It is arguable that, because solar energy is theoretically a very high temperature resource, we should try to harness it at this very high temperature for efficient conversion. We should then use the waste heat for low temperature purposes instead of downgrading the solar energy with low temperature collectors at the start. Such considerations might be important if solar energy were to be used for many purposes on a large scale.

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