Supplement to Chapter 12: Curve Fitting with Polynomials

12.1 Interpolation Polynomials

Exercises

1. The saturated vapor pressure of water is 0.611 kPa at 0°C, 1.227 kPa at 10°C, and 2.337 kPa at 20°C. Use polynomial interpolation to estimate the saturated vapor pressure of water at 5°C and 15°C. Check your result in Table 12.4.

2. Find the polynomial that fits the cube roots of the unequally spaced values of *x* in the table below.

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    Values of x: 0  1  8  27
Cube roots of x: 0  1  2   3
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Plot the graph of the polynomial and compare it with the graph of the cube roots of x = 0, 5, 10, 15, 20, 25, 30.

3. Use an efficient method to evaluate the polynomial

$$y = -0.3273 + 0.3656\, x - 0.04002\, x^2 + 0.001673\, x^3$$

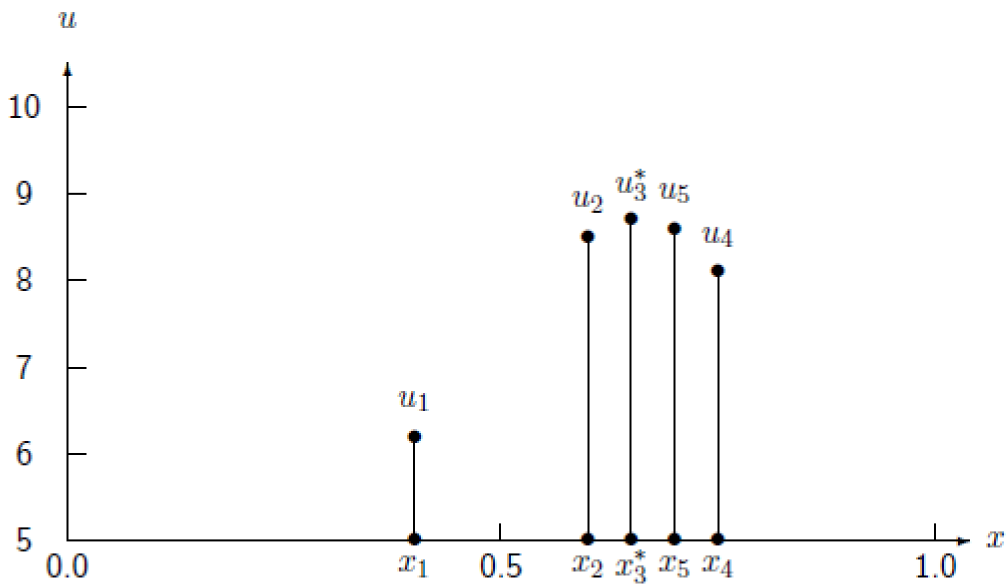
at the points x = 2.5, 5.5, 8.5. You can check your results approximately in Fig. 12.2.

Additional Topic: Simple Optimal Search

Optimization is the process of finding the "best" value of an objective function, such as the maximum yield in a chemical process, or the minimum cost of energy in the process.

The simplest optimization problem is to find the value x^* of x that gives the maximum value u^* of an objective function u = f(x) in an interval from x = a to b where there is just one maximum.

If there is no simple mathematical formula for the objective function, the optimum must be found instead from a series of laboratory experiments or runs of a computer model. Each experiment, or computer run, may give only one point on the graph of the objective function. When the work is expensive or time-consuming we want a method of searching for the optimum with the required accuracy using the fewest number of experiments or computer runs.



Example of a dichotomous search.

One simple method, called a *dichotomous search*, with pairs of experiments as shown in the following example. (See the figure above.)

Suppose we know that the maximum value u* is at a point x* between 0 and 1. It is best to begin with two experinemts near the center of the interval.

Let $x_1 = 0.4$ and $x_2 = 0.6$, and suppose that the experiments give $u_1 = 6.2$ and $u_2 = 8.5$. These results show that the maximum is between $x_1 = 0.4$ and x = 1.

We continue with two experiments near the center of the interval from x_1 to x = 1. Let $x_3 = 0.65$ and $x_4 = 0.75$, and suppose these experiments give $u_3 = 8.7$ and $u_4 = 8.1$. This shows that the maximum is between $x_2 = 0.6$ and $x_4 = 0.75$.

We may continue with one more experiment at $x_5 = 0.7$. Suppose this gives $u_5 = 8.6$. Our final conclusion is that the maximum is near $x^*_3 = 0.65$, and it has a value near $u^*_3 = 8.7$.

Exercise

Suppose the objective function is $u = 10\ x^{1.5}\ \exp(-x^{12.5})$. Use the method of searching with pairs to find x* in the interval 0 to 1 that gives the maximum u* at a precision of two decimal digits in x and u.
