Research Methodology Textbook

Supplement to Chapters 7 and 8 Numerical Measurements: Units and Dimensions

7.2 Significant Digits

Exercises

How many significant digits are there in the following quantities? (a) 3.5 m, (b) 4001 kg, (c) 10200 km, (d) 0.0023 s.

Assume that the numbers in the following expressions are accurate except for possible errors in the last digit given. Calculate the numerical values of these expressions and give your result so that they are accurate except for possible errors in the last digit in your answer.

1. $37.33 + 2.666$
2. $0.056 \times 10^{2} + 11.8 \times 10^{-1}$
3. 12.765×0.200
4. $(9.44 \times 10^{4}) \times (2.7722 \times 10^{12})$
5. $(4.865 \times 10^{10})/(0.020 \times 10^{4})$
6. $(12.45 - 11.65) \times 2.68$

7. Determine the changes in the quantities (a) $(36.479 \times 2.6)/14.85$, and (b) $17.524 + 2.4 - 3.56$, by unit increases in the least significant digit of each number in the expressions. Discuss how these changes affect the number of significant digits in the results.

8. A mathematical handbook gives the following polynomial approximation *p(x)* to the trigonometric function sin *x* from *x* = 0 to *x* = π/2:

$$p(x) = x - 0.16605x^{3} + 0.00761x^{5}.$$

Use your calculator to find the errors *e(x)* in *p(x)* given by

$$e(x) = p(x) - \sin x$$

as a function of x. You can do this by calculating numerical values of $e(x)$ for $x = 0$, 0.1π, 0.2π, 0.3π, 0.4π, 0.5π. How many decimal digits given by the formula $p(x)$ can be used with confidence?

8.1 The International System of Units

You can download a concise summary of the International System of Units by the *Bureau International des Poids et Mesures* at
https://www.bipm.org/utils/common/pdf/si_summary_en.pdf

Exercises

1. The energy E of an oscillating body of mass m is given by

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2,$$

where v is the velocity of the body, x is the displacement of the body from its equilibrium position, and k is a constant. Derive the basic SI units of measure of the constant k.

2. The rate of heat conduction dQ/dt along a rod of cross section area A in which the temperature gradient is dT/dx, is given by

$$dQ/dt = -kA\, dT/dx,$$

where k is the thermal conductivity. Prove that the SI units of measure of k can be written $\mathrm{W\ m^{-1}\ K^{-1}}$.

3. Look up the meanings of the following commonly used names and symbols and express the values in SI units with appropriate prefixes: 700 tonnes, 250 hectares, 5 bars, 850 hPa, 10000 lx.

8.3 Calculations with Measurements

8.3.2 Multiplication and Division (page 54)

According to the official recommendation, the examples in this section should be written as follows:

1. Suppose $s = vt$, where s is the distance traveled by a body moving at a velocity v in a time t. If $v = 4.6$ m/s and $t = 30$ s, then

$$s = 4.6\ \mathrm{m/s} \times 30\ \mathrm{s} = (4.6 \times 30)(\mathrm{ms^{-1}s}) = 138\ \mathrm{m}.$$

2. The height h of a column of mercury of density $\rho = 13.6 \times 10^3$ kg/m^3 supported by a pressure p of one atmosphere (1.013×10^5 Pa) is given by $p/(\rho g)$, where g is the acceleration of gravity (9.80 m/s^2). Then:

$$h = p/(\rho g)$$
$$= (1.013 \times 10^5 \text{ N/m}^2)/(13.6 \times 10^3 \text{ kg m}^{-3} \times 9.80 \text{ m s}^{-2})$$
$$= (1.013 \times 10^5)(13.6 \times 10^3 \times 9.80)^{-1}(\text{m kg s}^{-2}\text{ m}^{-2})(\text{kg m}^{-3})^{-1}(\text{m s}^{-2})^{-1}$$
$$= 0.760 \text{ m.}$$

Exercises

In the exercises below write out the calculations showing the units and give the final numerical results with an appropriate number of significant digits.

1. The periodic time T of a pendulum of length L is given by

$$T = 2\pi\sqrt{(L/g)},$$

where g is the acceleration of gravity. Calculate the value of g for measurements in which 100 swings last 201 s and the length of the pendulum is 1.00 m.

2. The heat capacity C of a metal sample at the very low temperature $T = 0.5$ K is 5 mJ/K, and at $T = 1.0$ K the heat capacity is 1.8 mJ/K. Theory states that C is a function of T given by

$$C = aT + b/T^2,$$

where a and b are constants. Calculate a and b from the data. [Use the equation $CT^2 = aT^3 + b$.]

3. Suppose that at height $z_1 = 2$ m above the ground the wind speed is $u_1 = 2.9$ m/s, and at height $z_2 = 10$ m above the ground the wind speed is $u_2 = 4.2$ m/s, and the wind speeds satisfy the equation

$$u_2/u_1 = (z_2/z_1)^k,$$

where k is a constant. Find k and use it to calculate the wind speed at height 30 m above the ground.

8.4 Dimensional Analysis

Do the two exercises in Section **8.4.2 Exercises**, page 56, and the following exercise on Section **8.4.3 Dimensionless Variables**.

Exercise

The *heat removal factor* of a solar water heater is a function of four variables:

- The mass flow rate m of water through the collector (kg s^{-1}).
- The specific heat capacity C_p of the water (J kg^{-1} K^{-1}).
- The area A of the collector (m^2).
- The overall heat loss coefficient U of the collector (W m^{-2} K^{-1}).

Find a dimensionless product of these four variables.
